Probability and Statistics in Engineering, Fall 2016

Exercise #2

1. Which of the following events are equal? Why?

   A = {1, 3};

   B = {x | x is a number on a die};

   C = {x | $x^2 - 4x + 3 = 0$ };

   D = {x | x is the number of heads when six coins are tossed}.

Sol:

(a) $A = \{1, 3\}$.

(b) $B = \{1, 2, 3, 4, 5, 6\}$.

(c) $C = \{x \mid x^2 - 4x + 3 = 0\} = \{x \mid (x-1)(x-3) = 0\} = \{1, 3\}$.

(d) $D = \{0, 1, 2, 3, 4, 5, 6\}$. Clearly, $A = C$.

2. Solve the following questions.

   (a) In how many ways can 6 people be lined up to get on a bus?

   (b) If 3 specific persons, among 6, insist on following each other, how many ways are possible?

   (c) If 2 specific persons, among 6, refuse to follow each other, how many ways are possible?

Sol:

(a) By Theorem 2.3, there are $6! = 720$ ways.

(b) A certain 3 persons can follow each other in a line of 6 people in a specified order is 4 ways or in $(4)(3!) = 24$ ways with regard to order. The other 3 persons can then be placed in line in $3! = 6$ ways. By Theorem 2.1, there are total $(24)(6) = 144$ ways to line up 6 people with a certain 3 following each other.

(c) Similar as in (b), the number of ways that a specified 2 persons can follow each other in a line of 6 people is $(5)(2!)(4!) = 240$ ways. Therefore, there are $720 - 240 = 480$ ways if a certain 2 persons refuse to follow each other.

3. Given an electronic component. Let A be the event that the component fails a particular test and B be the event that the component displays strain but does not actually fail. Event A occurs with probability 0.20 and event B occurs with probability 0.35.

   (a) What is the probability that the component does not fail the test?

   (b) What is the probability that a component works perfectly well (i.e., neither displays strain nor fails the test)?

   (c) What is the probability that the component either fails or shows strain in the test?

Sol:

$P(A) = 0.2$ and $P(B) = 0.35$

(a) $P(A') = 1 - 0.2 = 0.8$;

(b) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.2 - 0.35 = 0.45$;

(c) $P(A \cup B) = 0.2 + 0.35 = 0.55$.

4. Prove that $P(A' \cap B') = 1 + P(A \cap B) - P(A) - P(B)$.

Sol:

$P(A' \cap B') = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B) = 1 + P(A \cap B) - P(A) - P(B)$.

5. The probability that a person visiting his dentist will have an X-ray is 0.6; the probability that a person who has an X-ray will also have a cavity filled is 0.3; and the probability that a person who has had an X-ray and a cavity filled will also have a tooth extracted is 0.1. What is the probability that a person visiting his dentist will have an X-ray, a cavity filled, and a tooth extracted?

Sol:

Consider the events:
X: a person has an X-ray,
C: a cavity is filled,
T: a tooth is extracted.
$P(X \cap C \cap T) = P(X)P(C \mid X)P(T \mid X \cap C) = (0.6)(0.3)(0.1) = 0.018$.

6. A regional telephone company operates three identical relay stations at different locations. During a one year period, the number of malfunctions reported by each station and the causes are shown below.

	A	B	C
Problems with electricity supplied	2	1	1
Computer malfunction	4	3	2
Malfunctioning electrical equipment	5	4	2
Caused by other human errors	7	7	5

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station C?

Sol:

Consider the events
E: a malfunction by other human errors,
A: station A, B: station B, and C: station C.
$P(C \mid E) = \frac{P(E \mid C)P(C)}{P(E \mid A)P(A)+P(E \mid B)P(B)+P(E \mid C)P(C)} = \frac{(5/10)(10/43)}{(7/18)(18/43)+(7/15)(15/43)+(5/10)(10/43)} = \frac{0.1163}{0.4419} = 0.2632$.