Probability and Statistics in Engineering, Fall 2016

<u>Exercise #3</u>

1. Suppose a special type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The pdf that characterizes the proportion Y that make a profit is given by

$$f(y)=\begin{cases} ky^4(1-y)^3, & 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

   (a) What is the value of k that renders the above a valid density function?
   (b) Find the probability that at most 50% of the firms make a profit in the first year.
   (c) Find the probability that a least 80% of the firms make a profit in the first year.

2. Magnetron tubes are produced from an automated assembly line. A sampling plan is used periodically to assess quality on the lengths of the tubes. This measurement is subject to uncertainty. It is thought that the probability that a random tube meets the length specification is 0.99. A sampling plan is used in which the lengths of 5 random tubes are measured. Y is defined as the number of tubes, out of 5, that meet the length specification.
   (a) Find the probability function of Y.
   (b) Suppose the result of a sampling plan is that 3 tubes are outside the specification. Use the probability function, obtained in (a), to support or refute the conjecture that the probability is 0.99 that a single tube meets the specification.

3. A candy company distributes boxes of chocolates with a mixture of creams, toffees, and cordials. Suppose that the weight of each box is 1 kilogram, but the individual weights of the creams, toffee, and cordials vary from box to box. For a randomly selected box, let X and Y represent the weights of the creams and the toffees, respectively. In addition, suppose that the joint density function of these variables is

$$f(x,y)=\begin{cases} 24xy, & 0 \le x \le 1,\ 0 \le y \le 1,\ x+y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

   (a) Find the probability that in a given box, the cordials account for more than 1/2 of the weight.
   (b) Find the marginal density for the weight of the creams.
   (c) Find the probability that the weight of the toffees in a box is less than 1/8 of a kilogram if it is know that the creams constitute 3/4 of the weight.

4. The amount of kerosene, in thousands of liters, in a tank at the beginning of any day is a random amount Y from which a random amount X is sold during that day. Suppose that the tank is not resupplied during the day so that $x \le y$, and assume that the joint density function of these

variables is

$$f(x,y)=\begin{cases}2, & 0<x<y<1\\ 0, & \text{elsewhere}\end{cases}$$

(a) Determine if X and Y are independent.

(b) Find $P(1/4<X<1/2 \mid Y=3/4)$.

5. Random variables X and Y represent the number of vehicles that arrive at 2 separate street corners during a certain 2-minute period. These street corners are fairly close together so it is important that traffic engineers deal with them jointly if necessary. The joint distribution of X and Y is known to be

$$f(x,y)=\frac{9}{16}\frac{1}{4^{x+y}}, \text{ for } x=0,1,2,\ldots, \text{ and } y=0,1,2,\ldots$$

(a) Are the two random variables X and Y independent? Explain the reason.

(b) What is the probability that, during the time period in question, less than 4 vehicles arrive at the two street corners?

6. Consider a system of components in which there are five independent components, each of which has an operational probability of 0.92. The system has a redundancy built in such that it does not fail if 3 out of the 5 components are operational. What is the probability that the total system is operational?