Probability and Statistics in Engineering, Fall 2016

Exercise #4

1. The amount of kerosene, in thousands of liters, in a tank at the beginning of any day is a random amount Y from which a random amount X is sold during that day. Suppose that the tank is not resupplied during the day so that $x \le y$, and assume that the joint density function of these variables is

$$f(x,y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the average amount of kerosene left in the tank at the end of a day.

Sol:

$E(Y - X) = \int_0^1 \int_0^y 2(y - x)\, dx\, dy = \int_0^1 y^2\, dy = \frac{1}{3}$. Therefore, the average amount of kerosene left in the tank at the end of each day is $(1/3)(1000) = 333$ liters.

2. A privately owned liquor store operates both a drive-in facility and a walk-in facility. On a randomly selected day, let X and Y, respectively, be the proportions of the time that the drive-in and walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x,y) = \begin{cases} \frac{2}{3}(x + 2y), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) What is the expected proportion of the time that the drive-in facility is in use?

(b) What the expected proportion of the time that the walk-in facility is in use?

(c) Find $E\left[\frac{X+Y}{2}\right]$.

(d) What is the covariance of X and Y?

(e) What is the correlation coefficient of X and Y?

Sol:

It is known $g(x) = \frac{2}{3}(x + 1)$, for $0 < x < 1$, and $h(y) = \frac{1}{3}(1 + 4y)$, for $0 < y < 1$.

(a) $\mu_X = \int_0^1 \frac{2}{3}x(x + 1)\, dx = \frac{5}{9}$ and $\mu_Y = \int_0^1 \frac{1}{3}y(1 + 4y)\, dy = \frac{11}{18}$.

(b) $E[(X + Y)/2] = \frac{1}{2}[E(X) + E(Y)] = \frac{7}{12}$.

3. In a support system in the U.S. space program, a single crucial component works only 85% of the time. In order to enhance the reliability of the system, it is decided that 3 components will be installed in parallel such that the system fails only if they all fail. Assume the components act independently and that they are equivalent in the sense that all 3 of them have an 85% success rate. Consider the random variable X as the number of components out of 3 that fail.

(a) Find the probability function for the random variable X.

(b) What is E(X)?

(c) What is Var(X)?

(d) What is the probability that the entire system is successful?

(e) What is the probability that the system fails?

(f) If the desire is to have the system be successful with probability 0.99, are three components sufficient? Why? If not, how many are required?

Sol:

(a) $f(x) = \binom{3}{x}(0.15)^x(0.85)^{3-x}$, for $x = 0, 1, 2, 3$.

x	0	1	2	3
$f(x)$	0.614125	0.325125	0.057375	0.003375

(b) $E(X) = 0.45$.

(c) $E(X^2) = 0.585$, so $Var(X) = 0.585 - 0.45^2 = 0.3825$.

(d) $P(X \leq 2) = 1 - P(X = 3) = 1 - 0.003375 = 0.996625$.

(e) 0.003375.

(f) Yes.