Probability and Statistics in Engineering, Fall 2016

Exercise #5

1. During a manufacturing process 15 units are randomly selected each day from the production line to check the percent defective. From historical information, it is known that the probability of a defective unit is 0.05. Any time that two or more defectives are found in the sample of 15, the process is stopped. This procedure is used to provide a signal in case the probability of a defective has increased.

   (a) What is the probability that on any given day the production process will be stopped?

   (b) Suppose that the probability of a defective has increased to 0.07. What is the probability that on any given day the production process will not be stopped?

Sol:

$n = 15$ and $p = 0.05$.

(a) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - \sum_{x=0}^{1} \binom{15}{x}(0.05)^x(1-0.05)^{15-x} = 1 - 0.8290 = 0.1710.$

(b) $p = 0.07$. So, $P(X \leq 1) = \sum_{x=0}^{1} \binom{15}{x}(0.07)^x(1-0.07)^{15-x} = 1 - 0.7168 = 0.2832.$

2. An electronic switching device occasionally malfunctions and may need to be replaced. It is known that the device is satisfactory if it makes, on the average, no more than 0.20 error per hour. A particular 5-hour period is chosen as a "test" on the device. If no more than 1 error occurs, the device is considered satisfactory.

   (a) What is the probability that a satisfactory device will be considered unsatisfactory on the basis of the test?

   (b) What is the probability that a device will be accepted as satisfactory when, in fact, the mean number of errors is 0.25?

Sol:

$\lambda = 0.2$, so $\lambda t = (0.2)(5) = 1$.

(a) $P(X \leq 1) = \sum_{x=0}^{1} \frac{e^{-1}(1)^x}{x!} = 2e^{-1} = 0.7358$. Hence, $P(X > 1) = 1 - 0.7358 = 0.2642$.

(b) $\lambda = 0.25$, so $\lambda t = 1.25$. $P(X|le1) = \sum_{x=0}^{1} \frac{e^{-1.25}(1.25)^x}{x!} = 0.6446$.

3. A production process produces items in lots of 50. Sampling plans exist in which lots are pulled aside periodically and exposed to a certain type of inspection. It is usually assumed that the proportion defective in the process is very small. It is also important to the company that lots containing defects be a rare event. Currently, the inspection plan by the company is to periodically sample randomly 10 out of the 50 in a lot, and if none are defective, no intervention

into the process is done.

(a) Suppose in a lot chosen at random, 2 out of 50 are defective. What is the probability that at least 1 in the sample of 10 from the lot is defective?

(b) From your answer in (a), comment about the quality of this sampling plan.

(c) What is the mean number of defects found out of 10?

Sol:

(a) $k = 2$; $P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{\binom{2}{0}\binom{48}{10}}{\binom{50}{10}} = 1 - 0.6367 = 0.3633$.

(b) Even though the lot contains 2 defectives, the probability of reject the lot is not very high. Perhaps more items should be sampled.

(c) $\mu = (10)(2/50) = 0.4$.

4. According to a study published by a group of sociologists, about two-thirds of the 20 million people in a country who take Valium are women. Assume that this figure is a valid estimate, find the probability that on a given day, the fifth prescription written by a doctor for Valium is

(a) the first prescribing Valium for a woman

(b) the third prescribing Valium for a woman

Sol:

(a) Using the geometric distribution, we have $g(5; 2/3) = (2/3)(1/3)^4 = 2/243$.

(b) Using the negative binomial distribution, we have

$$b^*(5; 3, 2/3) = \binom{4}{2}(2/3)^3(1/3)^2 = \frac{16}{81}.$$