Probability and Statistics in Engineering, Fall 2016

Exercise #6

1. A lawyer commutes daily from his suburban home to his midtown office. The average time for a one-way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume the distribution of trip times to be normally distributed.

   (a) What is the probability that a trip will take at least 1/2 hour?

   (b) If the office opens at 9:00 A.M. and he leaves his house at 8:45 A.M. daily, what percentage of the time is he late for work?

   (c) If he leaves the house at 8:35 A.M. and coffee is served at the office from 8:50 A.M. until 9:00 A.M., what is the probability that he misses coffee?

   (d) Find the length of time above which we find the slowest 15% of the trips.

   (e) Find the probability that 2 of the next 3 trips will take at least 1/2 hour.

Sol:

(a) $z = (30 - 24)/3.8 = 1.58$; $P(X > 30) = P(Z > 1.58) = 0.0571$.

(b) $z = (15 - 24)/3.8 = -2.37$; $P(X > 15) = P(Z > -2.37) = 0.9911$. He is late 99.11% of the time.

(c) $z = (25 - 24)/3.8 = 0.26$; $P(X > 25) = P(Z > 0.26) = 0.3974$.

(d) $z = 1.04$, $x = (3.8)(1.04) + 24 = 27.952$ minutes.

(e) Using the binomial distribution with $p = 0.0571$, we get $b(2; 3, 0.0571) = \binom{3}{2}(0.0571)^2(0.9429) = 0.0092$.

2. The heights of 1000 students are normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Assuming that the heights are recorded to the nearest half-centimeter, how many of these students would you expect to have heights

   (a) less than 160.0 centimeters?

   (b) between 171.5 and 182.0 centimeters inclusive?

   (c) equal to 175.0 centimeters?

   (d) greater than or equal to 188.0 centimeters?

Sol:

(a) $z = (159.75 - 174.5)/6.9 = -2.14$; $P(X < 159.75) = P(Z < -2.14) = 0.0162$. Therefore, $(1000)(0.0162) = 16$ students.

(b) $z_1 = (171.25 - 174.5)/6.9 = -0.47$, $z_2 = (182.25 - 174.5)/6.9 = 1.12$. $P(171.25 < X < 182.25) = P(-0.47 < Z < 1.12) = 0.8686 - 0.3192 = 0.5494$. Therefore, $(1000)(0.5494) = 549$ students.

(c) $z_1 = (174.75 - 174.5)/6.9 = 0.04$, $z_2 = (175.25 - 174.5)/6.9 = 0.11$. $P(174.75 < X < 175.25) = P(0.04 < Z < 0.11) = 0.5438 - 0.5160 = 0.0278$. Therefore, $(1000)(0.0278)$=28 students.

(d) $z = (187.75 - 174.5)/6.9 = 1.92$; $P(X > 187.75) = P(Z > 1.92) = 0.0274$. Therefore, $(1000)(0.0274) = 27$ students.

3. A commonly used practice of airline companies is to sell more tickets than actual seats to a particular flight because customers who buy tickets do not always show up for the flight. Suppose that the percentage of no-shows at flight time is 2%. For a particular flight with 197 seats, a total of 200 tickets were sold. What is the probability that the airline overbooked this flight? Use the normal-curve approximation to find the probability.

Sol:

$n = 200$; X = The number of no shows with $p = 0.02$. $z = \frac{3-0.5-4}{\sqrt{(200)(0.02)(0.98)}} = -0.76$.
Therefore, $P(\text{airline overbooks the flight}) = 1 - P(X \geq 3) \approx 1 - P(Z > -0.76) = 0.2236$.

4. Consider an electrical component failure rate of once every 5 hours. It is important to consider the time that it takes for 2 components to fail.

   (a) Assuming that the gamma distribution applies, what is the mean time that it takes for failure of 2 components?

   (b) What is the probability that 12 hours will elapse before 2 components fail?

Sol:

$1/\beta = 1/5$ hours with $\alpha = 2$ failures and $\beta = 5$ hours.

(a) $\alpha\beta = (2)(5) = 10$.

(b) $P(X \geq 12) = \int_{12}^{\infty} \frac{1}{5^2\Gamma(2)} x e^{-x/5}\, dx = \frac{1}{25}\int_{12}^{\infty} x e^{-x/5}\, dx = \left[-\frac{x}{5}e^{-x/5} - e^{-x/5}\right]_{12}^{\infty}$
$= 0.3084$.

5. The density function of the time Z in minutes between calls to an electrical supply store is:

$$f(z) = \begin{cases} \frac{1}{10} e^{\frac{-z}{10}} & , 0 < z < \infty \\ 0 & , \text{elsewhere} \end{cases}$$

   (a) What is the mean time between calls?

   (b) What is the variance in the time between calls?

   (c) What is the probability that the time between calls exceeds the mean?

Sol:

(a) $\mu = \frac{1}{10}\int_0^{\infty} z e^{-z/10}\, dz = -\, z e^{-z/10}\Big|_0^{\infty} + \int_0^{\infty} e^{-z/10}\, dz = 10$.

(b) Using integral by parts twice, we get

$$E(Z^2) = \frac{1}{10}\int_0^{\infty} z^2 e^{-z/10}\, dz = 200.$$

So, $\sigma^2 = E(Z^2) - \mu^2 = 200 - (10)^2 = 100$.

(c) $P(Z > 10) = -\, e^{z/10}\Big|_{10}^{\infty} = e^{-1} = 0.3679$.