Probability and Statistics in Engineering, Fall 2016

<u>Exercise #7</u>

1. The amount of time that a drive-through bank teller spends on a customer is a random variable with a mean $\mu = 3.2$ minutes and a standard deviation $\sigma = 1.6$ minutes. If a random sample of 64 customers is observed, find the probability that their mean time at the teller's counter is

   (a) at most 2.7 minutes;

   (b) more than 3.5 minutes;

   (c) at least 3.2 minutes but less than 3.4 minutes.

Sol:

$n = 64$, $\mu_{\bar{X}} = 3.2$, $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 1.6/8 = 0.2$.

(a) $z = (2.7 - 3.2)/0.2 = -2.5$, $P(\bar{X} < 2.7) = P(Z < -2.5) = 0.0062$.

(b) $z = (3.5 - 3.2)/0.2 = 1.5$, $P(\bar{X} > 3.5) = P(Z > 1.5) = 1 - 0.9332 = 0.0668$.

(c) $z_1 = (3.2 - 3.2)/0.2 = 0$, $z_2 = (3.4 - 3.2)/0.2 = 1.0$,
$P(3.2 < \bar{X} < 3.4) = P(0 < Z < 1.0) = 0.9413 - 0.5000 = 0.3413$.

2. The mean score for freshmen on an aptitude test at a certain college is 540, with a standard deviation of 50. What is the probability that two groups of students selected at random, consisting of 32 and 50 students, respectively, will differ in their mean scores by

   (a) more than 20 points?

   (b) an amount between 5 and 10 points?

   Assume the means to be measured to any degree of accuracy.

Sol:

$\mu_1 - \mu_2 = 0$, $\sigma_{\bar{X}_1 - \bar{X}_2} = 50\sqrt{1/32 + 1/50} = 11.319$.

(a) $z_1 = -20/11.319 = -1.77$, $z_2 = 20/11.319 = 1.77$, so
$P(|\bar{X}_1 - \bar{X}_2| > 20) = 2P(Z < -1.77) = (2)(0.0384) = 0.0768$.

(b) $z_1 = 5/11.319 = 0.44$ and $z_2 = 10/11.319 = 0.88$. So,
$P(-10 < \bar{X}_1 - \bar{X}_2 < -5) + P(5 < \bar{X}_1 - \bar{X}_2 < 10) = 2P(5 < \bar{X}_1 - \bar{X}_2 < 10) =$
$2P(0.44 < Z < 0.88) = 2(0.8106 - 0.6700) = 0.2812$.

3. The scores on a placement test given to college freshmen for the past five years are approximately normally distributed with a mean $\mu = 71$ and a variance $\sigma^2 = 8$. Would you still consider $\sigma^2 = 8$ to be a valid variance, if a random sample of 20 students who take this placement test this year obtain a value of $s^2 = 8$? Why?

Sol:

$\chi^2 = \frac{(19)(20)}{8} = 47.5$ while $\chi^2_{0.01} = 36.191$. Conclusion values are not valid.

4. Consider the following measurements of the heat producing capacity of the coal produced by two mines (in millions of calories per ton):

   Mine 1: 8260 8130 8350 8070 8340

   Mine 2: 7950 7890 7900 8140 7920 7840

   Can it be concluded that the two population variances are equal? Why?

Sol:

$s_1^2 = 15750$ and $s_2^2 = 10920$ which gives $f = 1.44$. Since, from Table A.6, $f_{0.05}(4, 5) = 5.19$, the probability of $F > 1.44$ is much bigger than 0.05, which means that the two variances may be considered equal. The actual probability of $F > 1.44$ is 0.3436 and $P(F < 1/1.44) + P(F > 1.44) = 0.7158$.

5. A taxi company tests a random sample of 10 steel-belted radial tires of a certain brand and recorded the following tread wear results:

   48000, 53000, 45000, 61000, 59000, 56000, 63000, 49000, 53000, and 54000 kilometers.

   If the population from which the sample was taken has population mean $\mu = 53000$ kilometers, does the sample information support that claim? Why?

Sol:

$\bar{x} = 54{,}100$ and $s = 5801.34$. Hence

$$t = \frac{54100 - 53000}{5801.34/\sqrt{10}} = 0.60.$$

So, $P(\bar{X} \geq 54,100) = P(T \geq 0.60)$ is a value between 0.20 and 0.30, which is not a rare event.